Compactifications and their Ordering

Given a $T_{3\frac{1}{2}}$ space (X, τ), (Y, f) is a T_2 compactification of X provided Y is a compact T_2 space and $f : X \to Y$ is an embedding with $f[X]$ dense in Y. Two compactifications (Y_1, f_1) and (Y_2, f_2) are related provided there is a homeomorphism $h : Y_1 \to Y_2$ such that $h \circ f_1 = f_2$. This is known to be an equivalence relation; a partial ordering of the equivalence classes is defined as follows: $[(Y_1, f_1)] \leq [(Y_2, f_2)]$ provided there is a continuous onto map $g : Y_2 \to Y_1$ such that $g \circ f_2 = f_1$. In this ordering the Stone-Cěch compactification is known to be the largest element and, if X is a non-compact locally compact space, that of the one-point compactification the smallest. The Stone-Cěch compactification and the one-point compactification are unique up to homeomorphism and will be denoted $(\beta X, \iota)$ and (X^+, ι^+).The Stone-Cěch compactification is characterized by an important extension property: Every continuous map from X into a compact, T_2 space Y has a unique extension to βX. If $f : X \to Y$ is the continous map, the extension will be denoted $\beta f : \beta X \to Y$, where $\beta f \circ \iota = f$.

Notice that the equivalence classes of compactifications are only classes but not sets. For example, any set of the appropriate cardinality can be topologized and supplied with an imbedding to make a compactification equivalent to the Stone-Cěch compactification. Since the collection of all sets of a fixed cardinality is a class but not a set, the equivalence class of the Stone-Cěch compactification is also not a set. When necessary, representatives of classes can be chosen to avoid this difficulty.

Detailed presentations of the above can be found in many introductory topology texts, e.g. Wilansky [1].

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http://www.susanjkleinart.com/compactification/

References

1. Wilansky, A., Topology for Analysis ,Ginn and Co., 1970.